SISTEMI DI DISEQUAZIONI

Si ha che $\left\{ x : \begin{cases} f(x) > 0 \\ g(x) > 0 \end{cases} \right\} \equiv \{ x : f(x) > 0 \text{ e } g(x) > 0 \}$

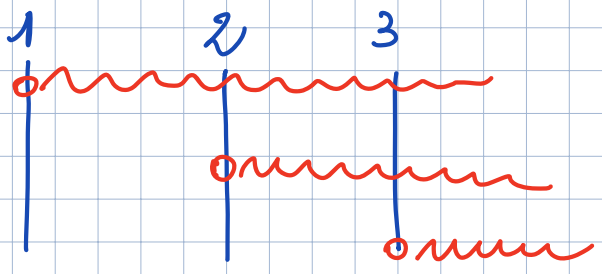
equivale a $\{ x : f(x) > 0 \} \cap \{ x : g(x) > 0 \}$.

ESEMPIO Per quali x $\begin{cases} x-1 > 0 \\ x-2 > 0 \\ x-3 > 0 \end{cases}$?

Dobbiamo calcolare l'intersezione fra

$$\{ x : \underset{x>1}{x-1>0} \} \cap \{ x : \underset{x>2}{x-2>0} \} \cap \{ x : \underset{x>3}{x-3>0} \}$$

$$(1; +\infty) \cap (2; +\infty) \cap (3; +\infty)$$



$$S = (3; +\infty)$$

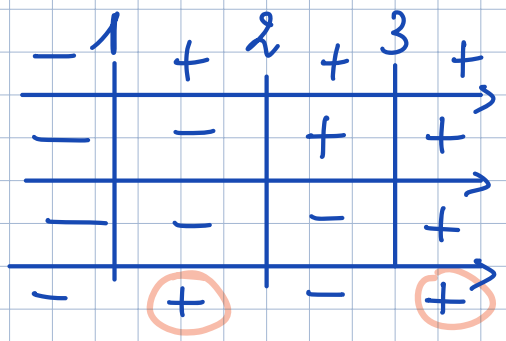
Oss: Confrontiamo le soluzioni di $\begin{cases} x-1 > 0 \\ x-2 > 0 \\ x-3 > 0 \end{cases}$

con le soluzioni di $(x-1)(x-2)(x-3) > 0$

$1^a > 0 \quad x > 1$

$2^a > 0 \quad x > 2$

$3^a > 0 \quad x > 3$



$S: \ (1;2) \cup (3;+\infty)$

Oss:

① $\{x: f(x) > g(x)\} \equiv \{x: f(x) - g(x) > 0\}$

② $\{x: f(x) > g(x)\} =$

$= \{x: \frac{f(x)}{g(x)} > 1 \ e \ g(x) > 0\} \cup \{x: \frac{f(x)}{g(x)} < 1 \ e \ g(x) < 0\}$

$\cup \{x: f(x) > 0 \ e \ g(x) = 0\}$

equivale a

$\left(\{x: \frac{f(x)}{g(x)} > 1\} \cap \{x: g(x) > 0\}\right) \cup \left(\{x: \frac{f(x)}{g(x)} < 1\} \cap \{x: g(x) < 0\}\right)$

$\cup \left(\{x: f(x) > 0\} \cap \{x: g(x) = 0\}\right)$

DISEQUAZIONI FRAZIONARIE

$$\left\{x : \frac{f(x)}{g(x)} \geq 0\right\} \equiv \left\{x : f(x) \geq 0 \text{ e } g(x) > 0\right\}$$

$$\cup \left\{x : f(x) \leq 0 \text{ e } g(x) < 0\right\}$$

ESEMPIO:

$$\frac{x-3}{2-3x} \geq 1$$

$$\frac{x-3}{2-3x} \geq 1 \Leftrightarrow \frac{x-3-2+3x}{2-3x} \geq 0 \Leftrightarrow \frac{4x-5}{2-3x} \geq 0$$

$$\left(\left\{x : 4x-5 \geq 0\right\} \cap \left\{x : 2-3x > 0\right\}\right) \cup \left(\left\{x : 4x-5 \leq 0\right\} \cap \left\{x : 2-3x < 0\right\}\right)$$

$$\left(\left[\frac{5}{4}; +\infty\right) \cap \left(-\infty; \frac{2}{3}\right)\right) \cup \left(\left(-\infty; \frac{5}{4}\right] \cap \left(\frac{2}{3}; +\infty\right)\right)$$

$$\emptyset \cup \left[\frac{2}{3}; \frac{5}{4}\right]$$

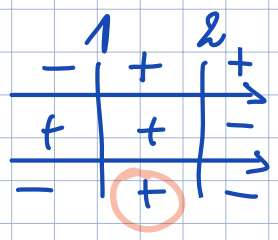
ESERCIZIO:

① $\left\{x : \frac{x-1}{2-x} > 0\right\} \equiv \left\{x : (x-1)(2-x) > 0\right\}$? ✓

② $\left\{x : \frac{x-1}{2-x} > 0\right\} \supset \left\{x : (x-1)(2-x) > 0\right\}$?

③ $\left\{x : \frac{x-1}{2-x} > 0\right\} \subset \left\{x : (x-1)(2-x) > 0\right\}$?

SOLUZIONE: $\frac{x-1}{2-x} > 0$ $N>0$ $x>1$ $D>0$ $x<2$



$1 < x < 2$

$(x-1)(2-x) > 0$

$(x-1)(x-2) < 0$ $\quad 1 < x < 2$

ESERCIZIO: $\frac{x-1}{x+1} - \frac{x+1}{x-1} < 2$

$$\frac{(x-1)^2 - (x+1)^2 - 2(x^2-1)}{(x+1)(x-1)} < 0$$

$$\frac{x^2-2x+1-(x^2+2x+1)-2x^2+2}{(x+1)(x-1)} < 0$$

$$\frac{\cancel{x^2}-2x+\cancel{1}-\cancel{x^2}-2x-\cancel{1}-2x^2+2}{(x+1)(x-1)} < 0$$

$$\frac{-2x^2-4x+2}{(x+1)(x-1)} < 0$$

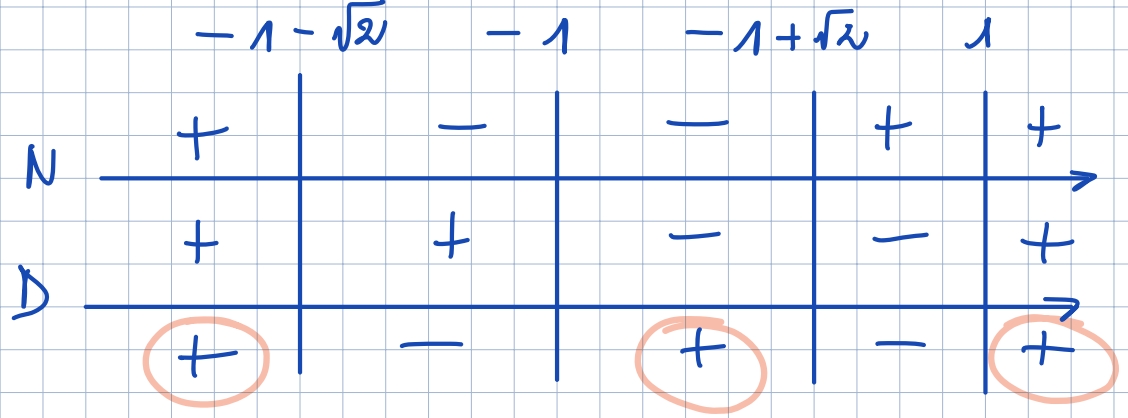
$$\frac{x^2+2x-1}{(x+1)(x-1)} > 0$$

$N > 0$ $\quad x^2+2x-1 > 0$

$\Delta = 4+4 = 8 = 2^3$ $\quad x_{1,2} \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$x < -1-\sqrt{2} \quad \vee \quad x > -1+\sqrt{2}$

$D > 0 \qquad (x+1)(x-1) > 0 \qquad x < -1 \vee x > 1$



$(-\infty; -1-\sqrt{2}) \cup (-1; -1+\sqrt{2}) \cup (1; +\infty)$

ESERCIZIO:

a) $\begin{cases} x^2 \geq x \\ x+3 < 9-x \end{cases}$

b) $\begin{cases} 1 < x^2 \leq 4 \\ x^2-5x+6 \geq 0 \end{cases}$

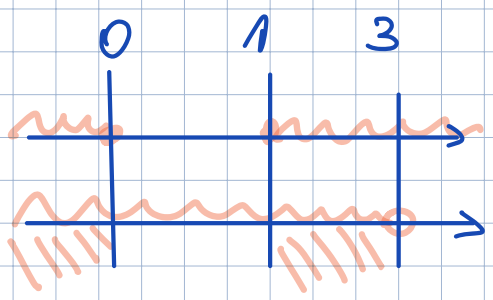
a) ① $x^2 - x \geq 0$

$x(x-1) \geq 0$

$x \leq 0 \quad \vee x \geq 1$

② $2x - 6 < 0$

$x < 3$

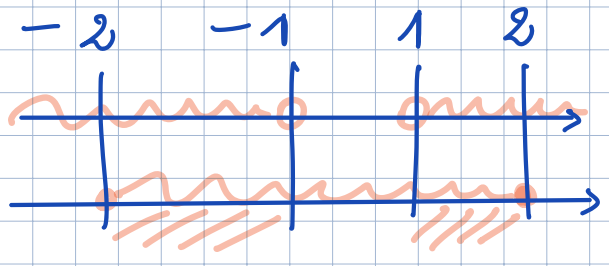


$x \leq 0 \quad \vee \quad 1 \leq x < 3$

$(-\infty; 0] \cup [1; 3)$

b) ① $1 < x^2 \leq 4$

$$\begin{cases} x^2 > 1 \\ x^2 \leq 4 \end{cases} \qquad \begin{cases} x < -1 \ \vee \ x > 1 \\ -2 \leq x \leq 2 \end{cases}$$



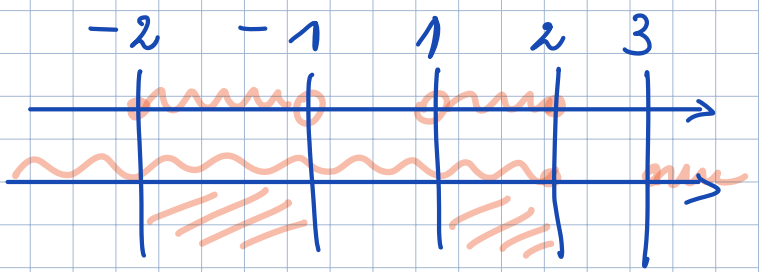
$-2 \leq x < -1 \quad \vee \quad 1 < x \leq 2$

$$\begin{cases} -2 \leq x < -1 \ \vee \ 1 < x \leq 2 \\ x^2 - 5x + 6 \geq 0 \end{cases}$$

② $(x-3)(x-2) \geq 0$

$x \leq 2 \ \vee \ x \geq 3$

$$\begin{cases} -2 \leq x < -1 \ \vee \ 1 < x \leq 2 \\ x \leq 2 \quad \vee \ x \geq 3 \end{cases}$$



$-2 \leq x < -1 \quad \vee \ 1 < x \leq 2$

$[-2; -1) \cup (1; 2]$

ESERCIZIO:

a) $P(x) = 6x^3 - 2ax + a^3x^2 - 1$

$P(x) = 6x^3 + a^3x^2 - 2ax - 1$

$D(x) = a - 2x$

$D(x) = -2x + a$

$$6x^3 + a^3x^2 - 2ax - 1 \quad \Big| \quad -2x + a$$
$$-6x^3 + 3ax^2$$
$$x^2(a^3+3a) - 2ax - 1$$
$$-x^2(a^3+3a) + \frac{1}{2}x(a^4+3a^2)$$
$$x\left(\frac{a^4}{2}+\frac{3}{2}a^2-2a\right) - 1$$
$$-x\left(\frac{a^4}{2}+\frac{3}{2}a^2-2a\right) + \frac{1}{2}\left(\frac{a^5}{2}+\frac{3}{2}a^3-2a^2\right)$$

$$-3x^2 - \frac{1}{2}x(a^3+3a) - \frac{1}{2}\underbrace{\left(\frac{a^4}{2}+\frac{3}{2}a^2-2a\right)}_{Q(x)}$$

$$R = \frac{a^5}{4} + \frac{3}{4}a^3 - a^2 - 1$$

$$P(x) = D(x) \cdot Q(x) + R$$

b) $P(x) = x^4 + 1$ $\qquad D(x) = x^2 - x\sqrt{2} + 1$

$$x^4 - - - 1 \quad \Big| \quad x^2 - x\sqrt{2} + 1$$
$$-x^4 + \sqrt{2}x^3 - x^2 \qquad\qquad x^2 + x\sqrt{2} + 1$$
$$x^3\sqrt{2} - x^2 + 1$$
$$-x^3\sqrt{2} + 2x^2 - x\sqrt{2}$$
$$x^2 - x\sqrt{2} + 1$$

$$R = 0$$