TANGENTE DELLA SOMMA

$$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}}$$

$\forall \alpha, \beta : \tan\alpha\cdot\tan\beta \neq 1$

DIM.

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} =$$

$$= \frac{\frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \frac{\sin\beta\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}}{\frac{\cancel{\cos\alpha}\cancel{\cos\beta}}{\cancel{\cos\alpha}\cancel{\cos\beta}} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

c.v.d.

FORMULA DI DUPLICAZIONE DELLA TANGENTE

$$\boxed{\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}}$$

FORMULE DI PROSTAFERESI

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

DIM.

$$\sin\alpha = \sin\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right) + \sin\left(\frac{\alpha-\beta}{2}\right)\cdot\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\sin\beta = \sin\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right) - \sin\left(\frac{\alpha-\beta}{2}\right)\cdot\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\boxed{\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$

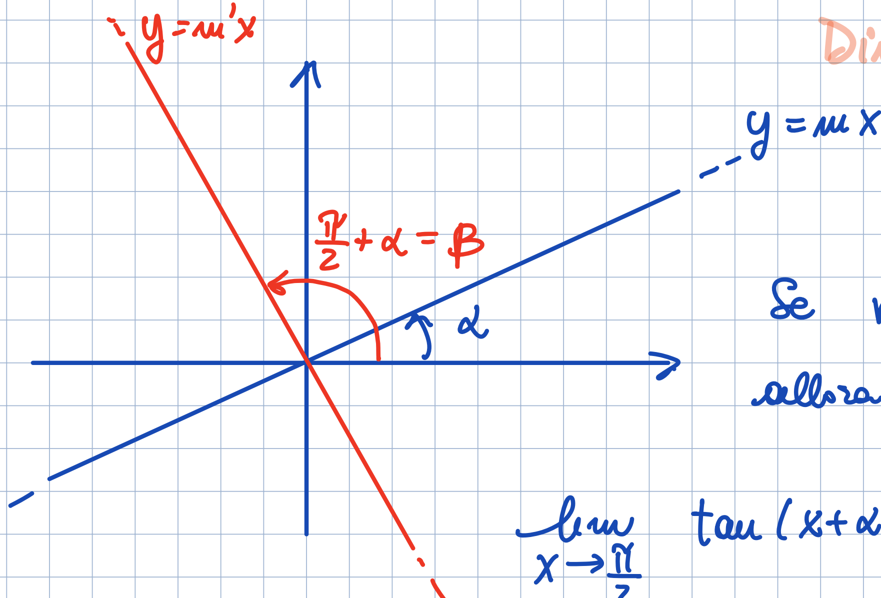
$$\boxed{\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$$

$$\cos\alpha = \cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right) - \sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\beta = \cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + \sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\boxed{\begin{aligned}\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\ \cos\alpha - \cos\beta &= -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\end{aligned}}$$

Oss: Date due rette di coefficienti angolari m ed m' queste sono $\perp$ se e solo se $m \cdot m' = -1$



Dim.

$m = \tan\alpha$

$m' = \tan\beta$

Se voglio che siano $\perp$ allora necessariamente $\beta = \frac{\pi}{2} + \alpha$

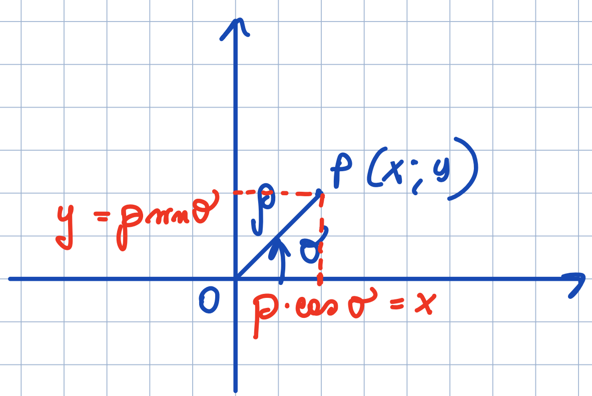
$$\lim_{x\to\frac{\pi}{2}} \tan(x+\alpha) = \lim_{x\to\frac{\pi}{2}} \frac{\tan x + \tan\alpha}{1 - \tan x \cdot \tan\alpha} =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cancel{\tan x}\left(1+\frac{\tan\alpha}{\tan x}\right)}{\cancel{\tan x}\left(\frac{1}{\tan x}-\tan\alpha\right)} = -\frac{1}{\tan\alpha} = -\frac{1}{m}$$

$$\frac{\tan\alpha}{\tan x} \to 0 \qquad \frac{1}{\tan x} \to 0$$

$\Rightarrow m \cdot m' = -1$ C. v. d.

COORDINATE POLARI



$P \in \mathbb{R}^2 \setminus \{0;0\}$

$\rho = d(P;O)$

θ = angolo misurato in senso antiorario formato da PO con l'asse delle x.

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

Viceversa $P(x;y)$ vogliamo trovare ρ e θ.

$\rho = \sqrt{x^2+y^2}$

$$\theta = \begin{cases} \arctan\frac{y}{x} & x>0 \text{ e } y\geq 0 \\ \frac{\pi}{2} & x=0 \text{ e } y>0 \\ \pi+\arctan\frac{y}{x} & x<0 \\ \frac{3}{2}\pi & x=0 \text{ e } y<0 \\ 2\pi+\arctan\frac{y}{x} & x>0 \text{ e } y<0 \end{cases}$$

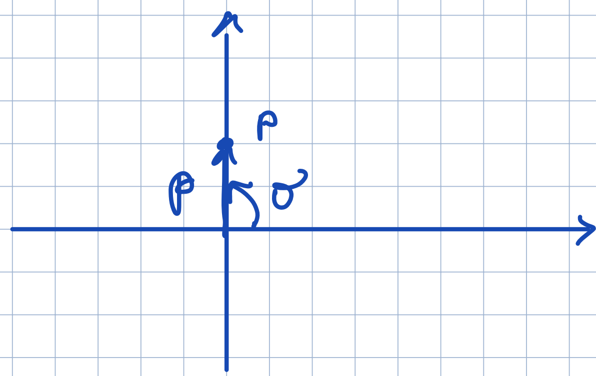
ESERCIZIO: Determinare le coordinate polari di

$(0;2)$ $(1;-1)$ $(-7\sqrt{3}, 7)$ $(-5;0)$

1) $(0;2)$

$\rho = \sqrt{0+4} = 2$

$\theta = \frac{\pi}{2}$

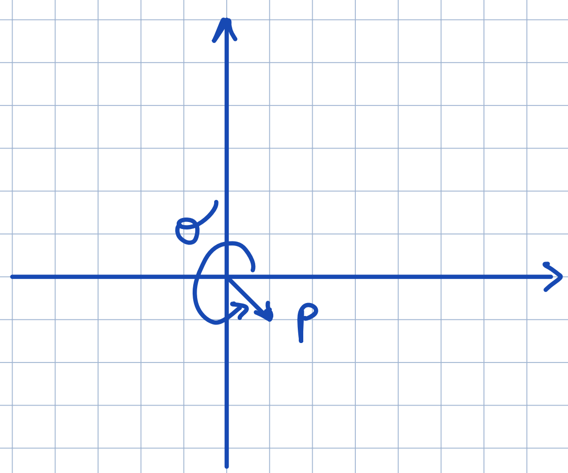


2) $(1;-1)$

$\rho = \sqrt{1+1} = \sqrt{2}$

$\theta = 2\pi + \arctan\left(\frac{y}{x}\right) = 2\pi + \arctan(-1)$

$= 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$



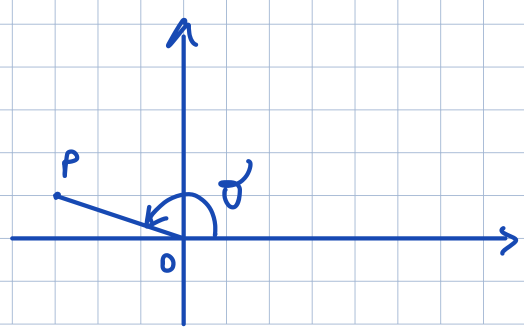
3) $(-7\sqrt{3};7)$

$\rho = \sqrt{196} = 14$

$\theta = \pi + \arctan\left(\frac{7}{-7\sqrt{3}}\right) =$

$= \pi + \arctan\left(-\frac{\sqrt{3}}{3}\right) =$

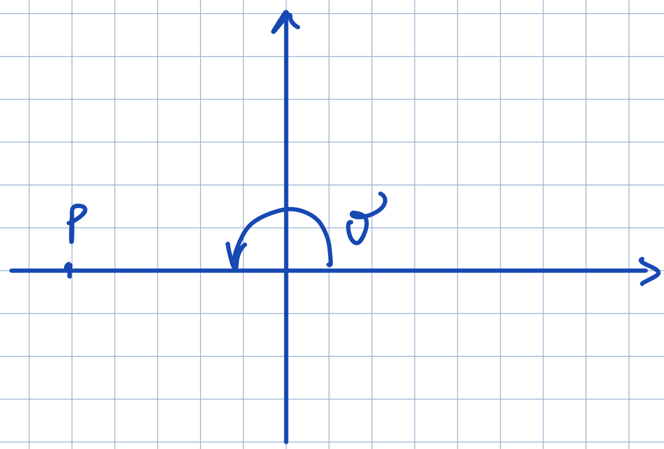
$= \pi - \frac{\pi}{6} = \frac{5}{6}\pi$



4) $(-5;0)$

$\rho = \sqrt{25+0} = 5$

$\theta = \pi$



ESERCIZIO: Determinare le coordinate cartesiane dei seguenti punti: $(2; \frac{\pi}{3})$ $(3; -3\pi)$ $(1; \frac{5}{4}\pi)$ $(6; \frac{23}{6}\pi)$

$$\left(\underset{\rho}{\underset{\downarrow}{2}}; \underset{\theta}{\underset{\downarrow}{\tfrac{\pi}{3}}}\right)$$

1) $(2; \frac{\pi}{3})$

$$x = \rho \cos\theta = 2 \cdot \cos\left(\frac{\pi}{3}\right) = 1$$

$$y = \rho \sin\theta = 2 \cdot \sin\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$(1; \sqrt{3})$$

2) $(3; -3\pi)$

$$x = 3\cos(-3\pi) = 3\cos(3\pi) = -3$$

$$y = 3\sin(-3\pi) = -3\sin(3\pi) = 0$$

$$(-3; 0)$$

3) $(1; \frac{5}{4}\pi)$

$$x = 1 \cdot \cos\left(\frac{5}{4}\pi\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$y = 1 \cdot \sin\left(\frac{5}{4}\pi\right) = \sin\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$(-\sqrt{2}/2; -\sqrt{2}/2)$$

4) $\left(6;\frac{23}{6}\pi\right)$

$$x = 6\cdot\cos\left(\frac{23}{6}\pi\right) = 6\cos\left(\frac{24-1}{6}\pi\right) = 6\cos\left(4\pi-\frac{\pi}{6}\right) = 6\cdot\frac{\sqrt{3}}{2} = 3\sqrt{3}$$

$$y = 6\sin\left(\frac{23}{6}\pi\right) = 6\cdot\sin\left(4\pi-\frac{\pi}{6}\right) = 6\left(-\frac{1}{2}\right) = -3$$

$$(3\sqrt{3};-3)$$

ESERCIZIO: Determinare seno, coseno e tangente di:

$-\frac{\pi}{6}$; $\frac{3}{4}\pi$; $\frac{7}{3}\pi$; $\frac{\pi}{4}+\frac{\pi}{3}$; $\frac{\pi}{3}-\frac{\pi}{4}$; $\frac{\pi}{8}$

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{+1/2}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\cos\left(\frac{3}{4}\pi\right) = \cos\left(\pi-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{3}{4}\pi\right) = \sin\left(\pi-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\tan\left(\frac{3}{4}\pi\right) = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$$

$$\cos\left(\frac{7}{3}\pi\right) = \cos\left(2\pi+\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{7}{3}\pi\right) = \sin\left(2\pi+\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{7}{3}\pi\right) = \sqrt{3}$$

$$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} =$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} =$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \frac{\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}}{\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}}\;\frac{(\sqrt{2}+\sqrt{6})}{(\sqrt{2}+\sqrt{6})} =$$

$$= \frac{2+6+2\sqrt{12}}{2-6} = \frac{8+4\sqrt{3}}{-4} =$$

$$= -2-\sqrt{3}$$

$$\sin\left(\frac{\pi}{8}\right) = \sin\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1-\cos(\pi/4)}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \frac{1}{2}\sqrt{2-\sqrt{2}}$$

$$\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$$

$$\tan\left(\frac{\pi}{8}\right) = \frac{\frac{1}{2}\sqrt{2-\sqrt{2}}}{\frac{1}{2}\sqrt{2+\sqrt{2}}} = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}} < 1 = \tan\frac{\pi}{4}$$