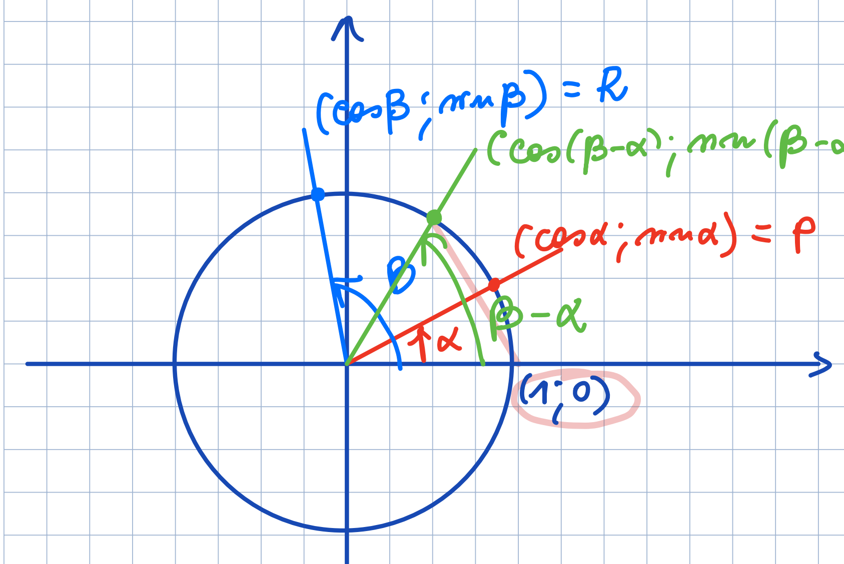
FORMULE DI ADDIZIONE / SOTTRAZIONE

$$\boxed{\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{aligned}}$$

DIM.

Proviamo che $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



$$d(Q;(1;0)) = \sqrt{(\cos(\beta-\alpha)-1)^2 + (\sin(\beta-\alpha)-0)^2}$$

$$d(R;P) = \sqrt{(\cos\beta-\cos\alpha)^2 + (\sin\beta-\sin\alpha)^2}$$

$$d(Q;(1;0)) \equiv d(R;P)$$

$$\sqrt{(\cos(\beta-\alpha)-1)^2 + (\sin(\beta-\alpha)-0)^2} = \sqrt{(\cos\beta-\cos\alpha)^2 + (\sin\beta-\sin\alpha)^2}$$

$$(\cos(\beta-\alpha)-1)^2 + (\sin^2 \beta-\alpha) = (\cos\beta-\cos\alpha)^2 + (\sin\beta-\sin\alpha)^2$$

$$\cos^2(\beta-\alpha) - 2\cos(\beta-\alpha) + 1 + \sin^2(\beta-\alpha) = \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha$$

$$\cancel{1} - 2\cos(\beta-\alpha) + \cancel{1} = \cancel{1} - 2\cos\beta\cos\alpha + \cancel{1} - 2\sin\beta\sin\alpha$$

$$-2\cos(\beta-\alpha) = -2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$$

$$\boxed{\cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha}$$

Da questa formula, posto $\beta = y$ e $\alpha = -x$

$$\boxed{\cos(y+x)} = \cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha =$$

$$= \cos y\cos(-x) + \sin y\sin(-x)$$

$$= \boxed{\cos y\cos x - \sin y\sin x}$$

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$$\boxed{\sin(y+x)} = \cos\left(y+x-\frac{\pi}{2}\right) = \cos y\cdot\cos\left(x-\frac{\pi}{2}\right) - \sin y\sin\left(x-\frac{\pi}{2}\right) =$$

$$= \cos y\sin x - \sin y\left(-\cos(x)\right) =$$

$$= \boxed{\cos y\sin x + \sin y\cos x}$$

$y = \beta \qquad x = -\alpha$

$$\boxed{\sin(\beta-\alpha)} = \sin(y+x) = \sin x\cos y + \sin y\cos x =$$

$$= \sin(-\alpha)\cos\beta + \sin\beta\cos(-\alpha) =$$

$$= -\sin\alpha\cos\beta + \sin\beta\cos(\alpha) =$$

$$= \boxed{\sin\beta\cos\alpha - \sin\alpha\cos\beta}$$

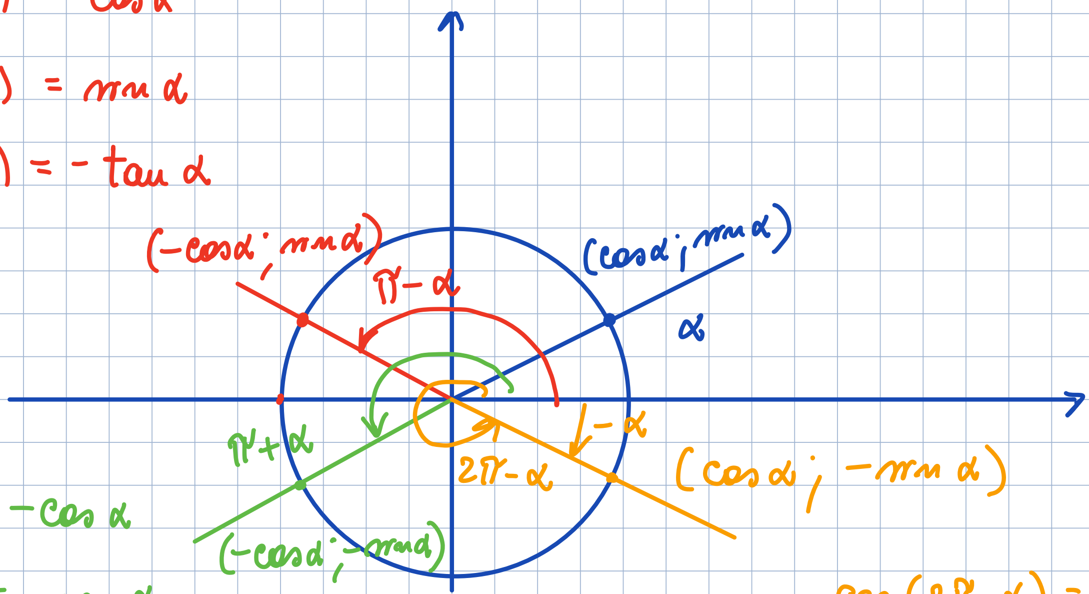
c.v.d.

ARCHI ASSOCIATI

$\cos(\pi-\alpha) = -\cos\alpha$

$\sin(\pi-\alpha) = \sin\alpha$

$\tan(\pi-\alpha) = -\tan\alpha$



$\cos(\pi+\alpha) = -\cos\alpha$

$\sin(\pi+\alpha) = -\sin\alpha$

$\tan(\pi+\alpha) = \tan\alpha$

$\cos(2\pi-\alpha) = \cos\alpha$

$\sin(2\pi-\alpha) = -\sin\alpha$

$\tan(2\pi+\alpha) = -\tan\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$

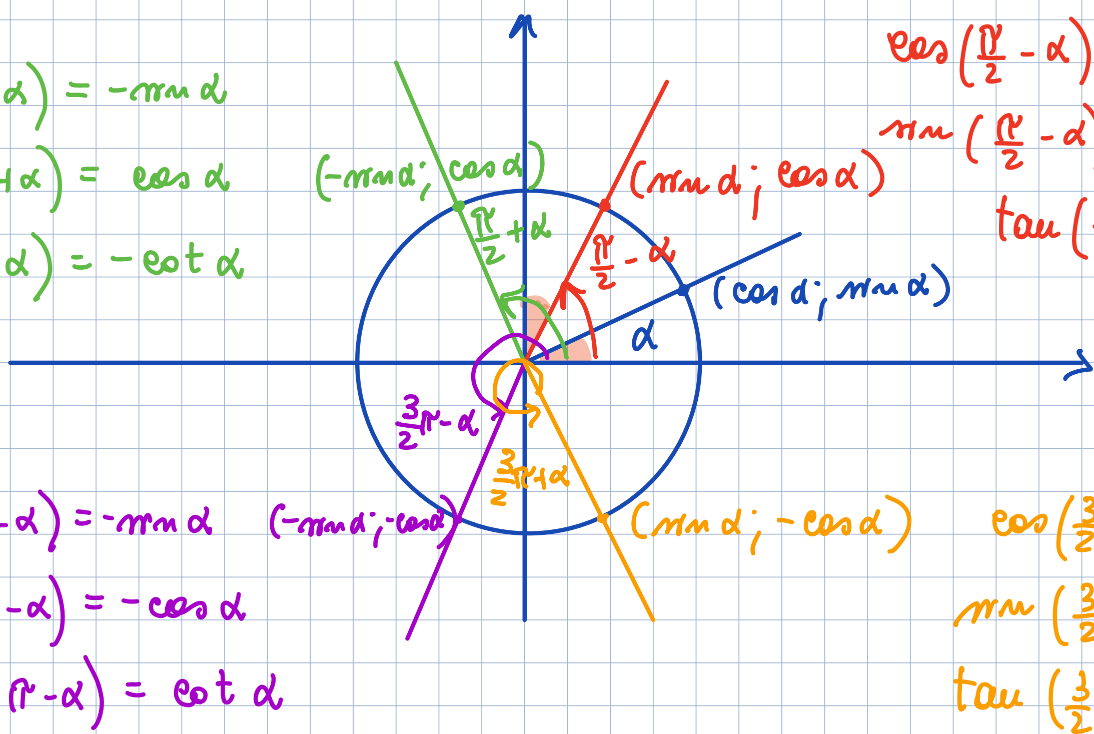
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$

$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$

$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$

$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$



$\cos\left(\frac{3}{2}\pi-\alpha\right) = -\sin\alpha$

$\sin\left(\frac{3}{2}\pi-\alpha\right) = -\cos\alpha$

$\tan\left(\frac{3}{2}\pi-\alpha\right) = \cot\alpha$

$\cos\left(\frac{3}{2}\pi+\alpha\right) = \sin\alpha$

$\sin\left(\frac{3}{2}\pi+\alpha\right) = -\cos\alpha$

$\tan\left(\frac{3}{2}\pi+\alpha\right) = -\cot\alpha$

FORMULE DI DUPLICAZIONE

RELAZIONE FONDAMENTALE

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

$$\boxed{\begin{aligned} \sin(2x) &= 2\sin x \cos x \\ \cos(2x) &= \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 \end{aligned}}$$

DIM.

$$\boxed{\sin(2x)} = \sin(x+x) = \sin x \cos x + \cos x \sin x = \boxed{2\sin x \cos x}$$

$$\boxed{\cos(2x)} = \cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x = \boxed{\cos^2 x - \sin^2 x}$$

FORMULE DI BISEZIONE

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \qquad \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

DIM.

$$\underline{\cos x} = \cos\left(2 \cdot \frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \underline{1 - 2\sin^2\frac{x}{2}}$$

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\boxed{\sin\frac{x}{2} = \begin{cases} +\sqrt{\frac{1-\cos x}{2}} & 2k\pi \le \frac{x}{2} \le (2k+1)\pi \quad k \in \mathbb{Z} \\ -\sqrt{\frac{1-\cos x}{2}} & (2k+1)\pi < \frac{x}{2} < (2k+2)\pi \end{cases}}$$

analogamente $\cos x = \cos\left(2\cdot\frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 2\cos^2\frac{x}{2} - 1$

$$\cos x = 2\cos^2\frac{x}{2} - 1$$

$$\cos^2\frac{x}{2} = \frac{\cos x + 1}{2}$$

$$\boxed{\cos\frac{x}{2} = \begin{cases} +\sqrt{\dfrac{1+\cos x}{2}} & -\dfrac{\pi}{2} + 2k\pi \le \dfrac{x}{2} \le \dfrac{\pi}{2} + 2k\pi \\ -\sqrt{\dfrac{1+\cos x}{2}} & \dfrac{\pi}{2} + 2k\pi < \dfrac{x}{2} < \dfrac{3}{2}\pi + 2k\pi \end{cases} \quad k \in \mathbb{Z}}$$

ESERCIZIO: Calcolare $\sin\frac{\pi}{12}$ e $\cos\frac{\pi}{12}$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1-\cos\pi/6}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} =$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} =$$

$$\boxed{\begin{aligned} \sqrt{a+\sqrt{b}} &= \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}} \\ \sqrt{a-\sqrt{b}} &= \sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}} \end{aligned}}$$

$$= \frac{1}{2}\left(\sqrt{\frac{2+\sqrt{4-3}}{2}} - \sqrt{\frac{2-\sqrt{4-3}}{2}}\right) = \frac{1}{2}\left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1+\cos\pi/6}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$= \frac{1}{2}\left(\sqrt{\frac{2+\sqrt{4-3}}{2}} + \sqrt{\frac{2-\sqrt{4-3}}{2}}\right) = \frac{1}{2}\left(\frac{\sqrt{6}+\sqrt{2}}{2}\right)$$

• ESPRIMERE FUNZIONI TRIGONOMETRICHE IN FUNZIONE DI SIN X (COS X / TAN X)

IN FUNZIONE DI SIN X

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \begin{cases} \sqrt{1-\sin^2 x} & -\frac{\pi}{2}+2k\pi \le x \le \frac{\pi}{2}+2k\pi \\ -\sqrt{1-\sin^2 x} & \frac{\pi}{2}+2k\pi < x < \frac{3}{2}\pi+2k\pi \end{cases}$$

$$\tan x = \frac{\sin x}{\cos x} = \begin{cases} \sqrt{\frac{\sin^2 x}{1-\sin^2 x}} & \text{se } \tan x > 0 \\ -\sqrt{\frac{\sin^2 x}{1-\sin^2 x}} & \text{se } \tan x < 0 \end{cases}$$

IN FUNZIONE DI COS X

$$\sin x = \begin{cases} \sqrt{1-\cos^2 x} & 2k\pi \le x \le \pi+2k\pi \\ -\sqrt{1-\cos^2 x} & \pi+2k\pi < x < 2\pi+2k\pi \end{cases}$$

$$\tan x = \frac{\sin x}{\cos x} = \begin{cases} \sqrt{\frac{1-\cos^2 x}{\cos^2 x}} & \tan x > 0 \\ -\sqrt{\frac{1-\cos^2 x}{\cos^2 x}} & \tan x < 0 \end{cases}$$

IN FUNZIONE DI TAN X

$$\sin x = \frac{\sin x}{\sqrt{\cos^2 x + \sin^2 x}} = \begin{cases} \sqrt{\dfrac{\sin^2 x \cdot \frac{1}{\cos^2 x}}{(\cos^2 x + \sin^2 x) \cdot \frac{1}{\cos^2 x}}} \\ -\sqrt{\dfrac{\sin^2 x}{\cos^2 x + \sin^2 x}} \end{cases} =$$

$$= \begin{cases} \sqrt{\dfrac{\tan^2 x}{1 + \tan^2 x}} & 0 + 2k\pi \le x \le \pi + 2k\pi \\ -\sqrt{\dfrac{\tan^2 x}{1 + \tan^2 x}} & \pi + 2k\pi < x < 2\pi + 2k\pi \end{cases}$$

$$\cos x = \frac{\cos x}{\sqrt{\cos^2 x + \sin^2 x}} = \begin{cases} \dfrac{1}{\sqrt{1+\tan^2 x}} & -\frac{\pi}{2} + 2k\pi \le x \le \frac{\pi}{2} + 2k\pi \\ -\dfrac{1}{\sqrt{1+\tan^2 x}} & \frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \end{cases}$$

FORMULE PARAMETRICHE

$t = \tan\left(\frac{x}{2}\right)$

$$\boxed{\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \tan x = \frac{2t}{1-t^2}}$$

DIM.

$$\sin x = \sin\left(2 \cdot \frac{x}{2}\right) = 2 \sin\frac{x}{2} \cos\frac{x}{2} = \frac{2 \sin\frac{x}{2} \cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}} = \frac{\frac{1}{\cos^2\frac{x}{2}}}{\frac{1}{\cos^2\frac{x}{2}}}$$

se $\cos^2\frac{x}{2} \neq 0$

$$= \frac{\dfrac{2 \sin\frac{x}{2} \cancel{\cos\frac{x}{2}}}{\cos^2\frac{x}{2}}}{\dfrac{\sin^2 x/2}{\cos^2 x/2} + \dfrac{\cancel{\cos^2 x/2}}{\cancel{\cos^2 x/2}}} = \frac{2 \tan\frac{x}{2}}{\tan^2\frac{x}{2} + 1} \underset{t = \tan\frac{x}{2}}{=} \frac{2t}{t^2+1}$$

$$\cos x = \cos\left(2\cdot\frac{x}{2}\right) = \cos^2\frac{x}{2} - \operatorname{sen}^2\frac{x}{2} = \frac{\cos^2\frac{x}{2} - \operatorname{sen}^2\frac{x}{2}}{\cos^2\frac{x}{2} + \operatorname{sen}^2\frac{x}{2}} \quad \frac{\frac{1}{\cos^2 x/2}}{\frac{1}{\cos^2 x/2}} =$$

se $\cos^2\frac{x}{2} \neq 0$

$$= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$$t = \tan\frac{x}{2}$$

$$\tan x = \frac{\operatorname{sen} x}{\cos x} = \frac{\frac{2t}{t^2+1}}{\frac{1-t^2}{1+t^2}} = \frac{2t}{\cancel{t^2+1}} \cdot \frac{\cancel{1+t^2}}{1-t^2} =$$

$$= \frac{2t}{1-t^2}$$

c.v.d.