LA FUNZIONE ESPONENZIALE

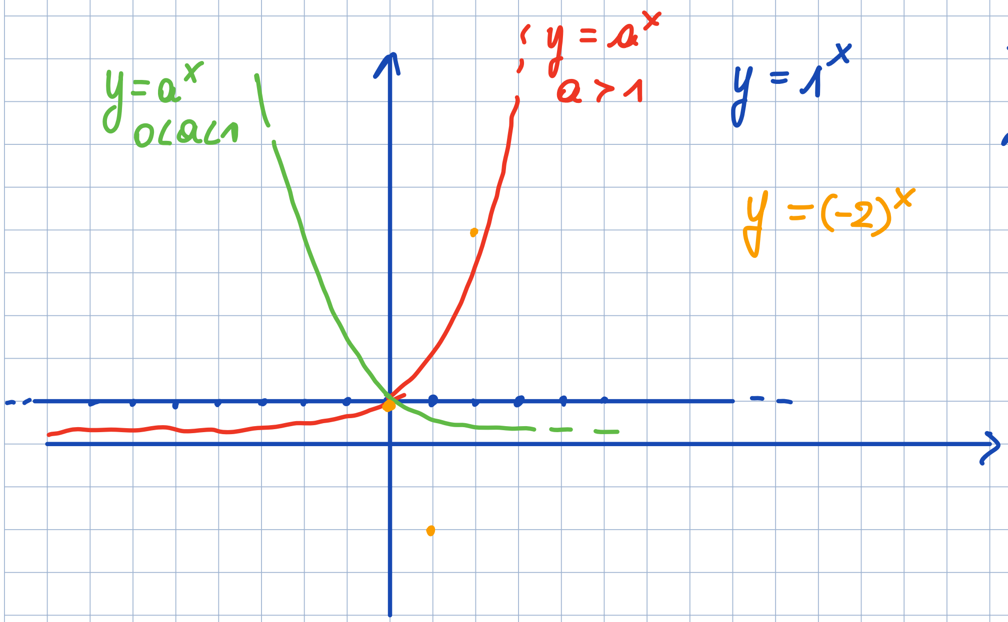
$$y = a^x$$

con $a > 0 \land a \neq 1$

- CONTINUA SU TUTTO $\mathbb{R}$

1) $a^x \cdot a^y = a^{x+y}$

2) $a^1 = a$



Oss. Il nome funzione esponenziale viene riservata ad a^x con $a = e \approx 2{,}718\ldots$ (numero di Nepero/ numero di Eulero)

PROPRIETÀ

3) $a^x > 0 \quad \forall x \in \mathbb{R}$ (es. $2^x > -7 \quad \forall x \in \mathbb{R}$)

4) $(a^x)^y = a^{x \cdot y} \quad \forall x \in \mathbb{R}$

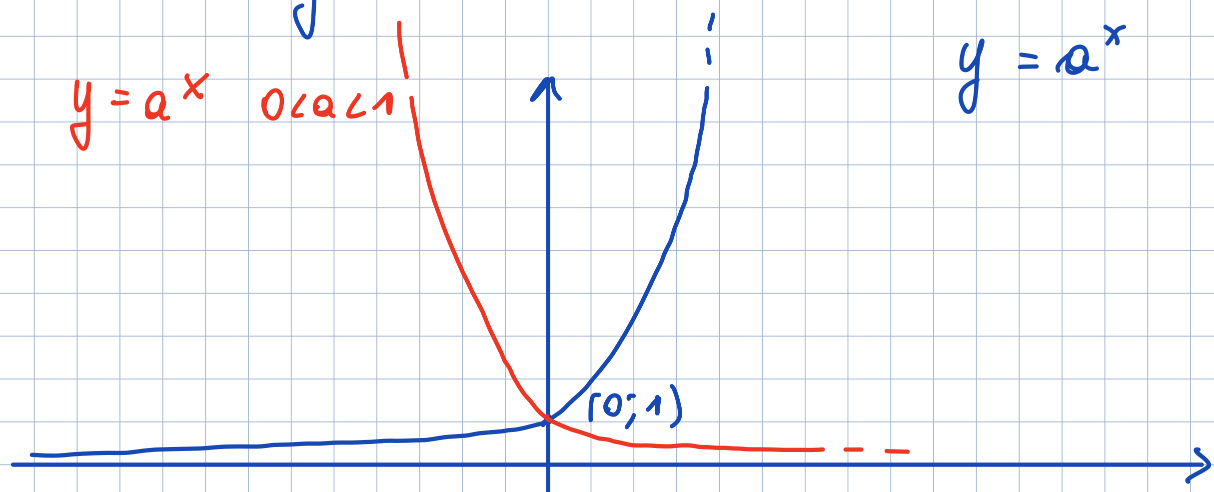
5) Sempre crescente se $a > 1$ e sempre decrescente se $0 < a < 1$

6) $a^0 = 1 \quad \forall a > 0$

7) $\sqrt[m]{a^n} = a^{\frac{n}{m}} \quad n, m \in \mathbb{Z}$

Oss. Dalla proprietà 4) e dalla 6) abbiamo che $a^{-x} = \left(\frac{1}{a}\right)^x$

ESEMPIO: Un materiale radioattivo soddisfa una legge di decadimento le cui soluzioni hanno un andamento esponenziale. $y = N_0 \cdot N^t$



- L'asse x è asintoto orizzontale della funzione $y = a^x$

- $\lim_{x \to +\infty} a^x = +\infty$

  $\lim_{x \to -\infty} a^x = 0$

- $\lim_{x \to +\infty} a^x = 0$

- $\lim_{x \to -\infty} a^x = +\infty$

ESERCIZIO:

a) $10^x = 100$ $\qquad 10^x = 10^2$ $\qquad x = 2$

b) $7^x = 1$ $\qquad 7^x = 7^0$ $\qquad x = 0$

c) $9^{x+2} = \sqrt[3]{3^{x+7}}$

d) $3 \cdot 5^x + 5^{x+1} = 8 \cdot 5^3$

e) $9^x - 3 = 2 \cdot 3^x$

f) $10^x + 10^{2-x} = 101$

SVOLGIMENTO

c) $(3^2)^{x+2} = 3^{\frac{x+7}{3}}$

$3^{2x+4} = 3^{\frac{x+7}{3}}$

$2x+4 = \frac{x+7}{3}$

$6x+12 = x+7$

$5x = -5$

$x = -1$

d) $3 \cdot 5^x + 5^{x+1} = 8 \cdot 5^3$

$5^x(3+5) = 8 \cdot 5^3$

$5^x \cdot \cancel{8} = \cancel{8} \cdot 5^3$

$5^x = 5^3$

$x = 3$

e) $9^x - 3 = 2 \cdot 3^x$

$3^{2x} - 2 \cdot 3^x - 3 = 0$

$(3^x)^2 - 2 \cdot 3^x - 3 = 0$

$3^x = t$

$t^2 - 2t - 3 = 0$

$\Delta = 4 + 12 = 16 = 4^2$

$t_{1,2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$

$3^x = 3 \longrightarrow x = 1$

$3^x = -1$ IMPOSSIBILE

f) $10^x + 10^{2-x} = 101$

$10^x = t$

$10^x + \frac{10^2}{10^x} = 101$

$t + \frac{100}{t} = 101$

$t^2 - 101t + 100 = 0$

$\Delta = (101)^2 - 400 = 10201 - 400 = 9801 = 99^2$

$t_{1,2} = \frac{101 \pm 99}{2} = \begin{cases} \frac{200}{2} = 100 \\ \frac{2}{2} = 1 \end{cases}$

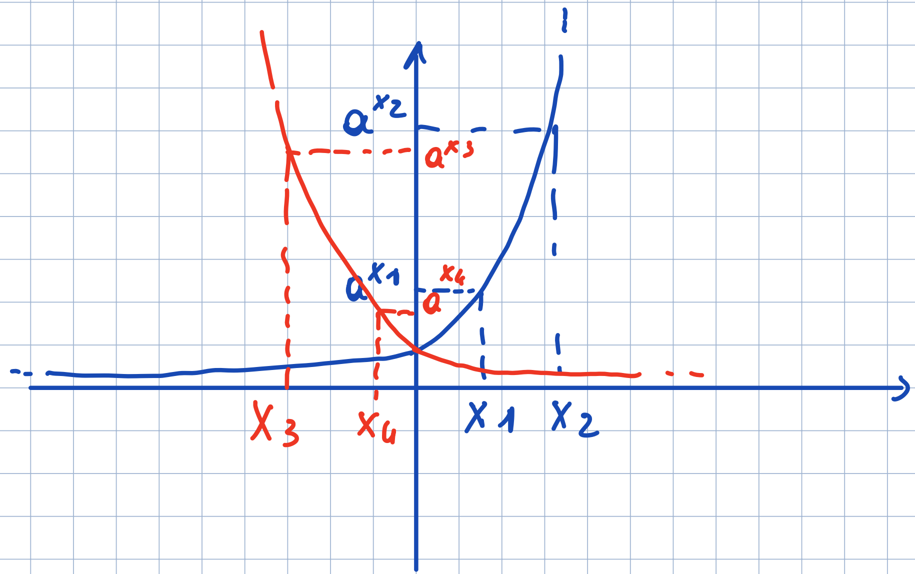
$10^x = 100$
$x = 2$

$10^x = 1$
$x = 0$

DISEQUAZIONI ESPONENZIALI

$y = a^x$ $\quad a > 1$

$0 < a < 1$



$$\boxed{\begin{array}{c} a^{x_1} < a^{x_2} \\ x_1 < x_2 \end{array}}$$

$$\boxed{\begin{array}{c} a^{x_3} > a^{x_4} \\ x_3 < x^4 \end{array}}$$

ESERCIZI:

a) $4^x \leq 32$

b) $\left(\frac{1}{4}\right)^{x-1} < 64$

c) $\left(\frac{1}{5}\right)^{2x+1} < 625$

d) $2^x \cdot 3^{x+1} \leq \frac{6^{3x}}{2}$

SVOLGIMENTO:

a) $4^x \leq 32 \qquad 2^{2x} \leq 2^5 \qquad 2x \leq 5 \qquad x \leq \frac{5}{2}$

b) $\left(\frac{1}{4}\right)^{x-1} < 64 \qquad 4^{-x+1} < 4^3 \qquad -x+1 < 3$

$x > -2$

c) $\left(\frac{1}{5}\right)^{2x+1} < 625 \qquad 5^{-2x-1} < 5^4 \qquad -2x-1 < 4$

$-1-4 < 2x$

$x > -\frac{5}{2}$

d) $2^x \cdot 3^{x+1} \leq \frac{6^{3x}}{2}$

$$2^x \cdot 3^x \cdot 3 \le 6^{3x} \cdot \frac{1}{2}$$

$$\frac{\cancel{6^x} \cdot 3}{\cancel{6^x}} \le \frac{6^{3x} \cdot 1/2}{6^x}$$

$$2 \cdot 3 \le 6^{2x} \cdot \frac{1}{\cancel{2}} \cdot \cancel{2}$$

$$6 \le 6^{2x}$$

$$1 \le 2x$$

$$x \ge 1/2$$

ESERCIZI:

a) $$\frac{2^x - 4}{1 - 3^x} > 0$$

b) $$45 \cdot 2^{2x-2} < -35 \cdot 4^{x-1}$$

c) $$\left(\frac{1}{4}\right)^x - 7 \cdot \left(\frac{1}{2}\right)^x - 8 \ge 0$$

d) $$\frac{\sqrt{3^{6x} : 3^2}}{3^7} < 3^{-x}$$

SVOLGIMENTO:

a) $N > 0 \quad 2^x - 4 > 0 \quad 2^x > 4 \quad x > 2$

$D > 0 \quad 1 - 3^x > 0 \quad 3^x < 1 \quad x < 0$

		0		2	
N	$-$		$-$		$+$
D	$+$		$-$		$-$
	$-$		$+$		$-$

$0 < x < 2$

b) $45 \cdot 2^{2x-2} < -35 \cdot 4^{x-1}$

$45 \cdot \frac{2^{2x}}{2^2} < -35 \cdot (2^2)^{(x-1)}$

$45 \cdot \frac{2^{2x}}{4} < -35 \cdot 2^{2x-2}$

$45 \cdot \frac{\cancel{2^{2x}}}{\cancel{4}} < -35 \cdot \frac{\cancel{2^{2x}}}{\cancel{4}}$

$45 < -35$ IMPOSSIBILE ($\nexists x \in \mathbb{R}$)

c) $\left(\frac{1}{4}\right)^x - 7 \cdot \left(\frac{1}{2}\right)^x - 8 \geq 0$

$\left(\frac{1}{2}\right)^{2x} - 7 \cdot \left(\frac{1}{2}\right)^x - 8 \geq 0$ $\qquad \left(\frac{1}{2}\right)^x = t$

$t^2 - 7t - 8 \geq 0$

$\Delta = 49 + 32 = 81 = 9^2$

$t_{1,2} = \frac{7 \pm 9}{2} = \begin{cases} 8 \\ -1 \end{cases}$

$t \leq -1 \quad \vee \quad t \geq 8$

$\left(\frac{1}{2}\right)^x \leq -1 \quad \vee \quad \left(\frac{1}{2}\right)^x \geq 8$

IMPOSSIBILE

$2^{-x} \geq 2^3$

$\boxed{x \leq -3}$

d) $\frac{\sqrt{3^{6x} : 3^2}}{3^7} < 3^{-x}$ $\qquad \frac{\left(3^{6x-2}\right)^{1/2}}{3^7} < 3^{-x}$

$$\frac{3^{3x-1}}{3^7} < 3^{-x}$$

$$3^{3x-1-7} < 3^{-x}$$

$$3x-8 < -x$$

$$4x < 8$$

$$x < 2$$

ESERCIZIO:

$$3^{2x} - 3^x - 5 = 0 \qquad 3^x = t$$

$$t^2 - t - 5 = 0$$

$$\Delta = 1 + 20 = 21$$

$$t_{1,2} = \frac{1 \pm \sqrt{21}}{2} = \begin{cases} \dfrac{1+\sqrt{21}}{2} \\ \dfrac{1-\sqrt{21}}{2} < 0 \end{cases}$$

$$3^x = \frac{1+\sqrt{21}}{2}$$

$$\log_3 3^{x} = \log_3\left(\frac{1+\sqrt{21}}{2}\right)$$

$$x \cdot \underbrace{\log_3 3}_{\to 1} = \log_3\left(\frac{1+\sqrt{21}}{2}\right)$$

$$x = \log_3\left(\frac{1+\sqrt{21}}{2}\right)$$