13/9/22

Equazioni e disequazioni trigonometriche

EQUAZIONI

1) EQ. GONIOMETRICHE ELEMENTARI $\quad -1 \le m \le 1, \ k \in \mathbb{Z}$

$\sin x = m$

$\Leftrightarrow \quad x = \alpha + 2k\pi \quad \vee \quad x = \pi - \alpha + 2k\pi,$

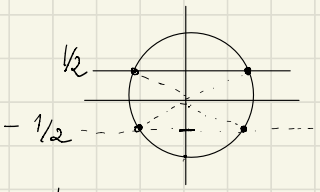
CON $\alpha = \arcsin(m)$.

$\cos x = m \quad \Leftrightarrow \quad x = \pm\alpha + 2k\pi, \quad \alpha = \arccos(m).$

$\tan x = m, \ m \in \mathbb{R} \quad \Leftrightarrow \quad x = \alpha + k\pi, \quad \alpha = \arctan(m).$

ES. $\sin x = \frac{1}{2} \quad \Leftrightarrow \quad x = \frac{\pi}{6} + 2k\pi \quad \vee \quad x = \frac{5}{6}\pi + 2k\pi,$

$k \in \mathbb{Z}$



ES. $|\sin x| = \frac{1}{2} \quad \Leftrightarrow \quad \sin x = \frac{1}{2} \quad \vee \quad \sin x = -\frac{1}{2}$

$\Leftrightarrow \quad x = \frac{\pi}{6} + 2k\pi \quad \vee \quad x = \frac{5}{6}\pi + 2k\pi \quad \vee$

$x = \pi + \frac{\pi}{6} + 2k\pi \quad \vee \quad x = -\frac{\pi}{6} + 2k\pi.$

$= \frac{11}{6}\pi + 2k\pi.$

2) EQ. CHE SI PRESENTANO COME UGUAGLIANZA DI FUNZIONI GONIOMETRICHE

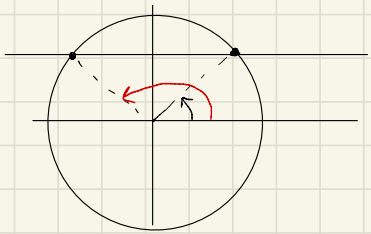
- $\sin f(x) = \sin g(x)$

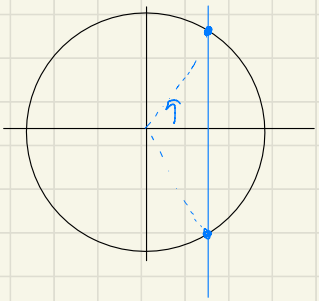
$$\Leftrightarrow \quad f(x) = g(x) + 2k\pi \quad \vee \quad f(x) = \pi - g(x) + 2k\pi, \quad k \in \mathbb{Z}$$

- $\cos f(x) = \cos g(x)$

$$\Leftrightarrow \quad f(x) = \pm g(x) + 2k\pi, \quad k \in \mathbb{Z}$$

- $\tan f(x) = \tan g(x) \quad \Leftrightarrow \quad f(x) = g(x) + k\pi, \quad k \in \mathbb{Z}$





ES. (a) $\tan(3x) = \tan\left(x + \frac{\pi}{2}\right)$

(b) $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{3}\right)$

(c) $\sin x = \sin\left(2x - \frac{\pi}{2}\right)$

(a) $3x = x + \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$

$$\Leftrightarrow \quad 2x = \frac{\pi}{2} + k\pi \quad \Leftrightarrow \quad x = \frac{1}{2}\left(\frac{\pi}{2} + k\pi\right)$$

$\Leftrightarrow \quad x = \frac{\pi}{4} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$

b) $\frac{\pi}{2} - x = x - \frac{\pi}{3} + 2k\pi \quad \vee \quad \frac{\pi}{2} - x = -\left(x - \frac{\pi}{3}\right) + 2k\pi$

$\Leftrightarrow \quad x = \frac{5}{12}\pi + k\pi, \; k \in \mathbb{Z} \quad \vee \quad \cancel{\frac{\pi}{2} = \frac{\pi}{3} + 2k\pi}$

IMPOS.

c) $x = 2x - \frac{\pi}{2} + 2k\pi \quad \vee \quad x = \pi - \left(2x - \frac{\pi}{2}\right) + 2k\pi, \; k \in \mathbb{Z}$

$\Leftrightarrow \quad x = \frac{\pi}{2} + 2k\pi \quad \vee \quad x = \frac{\pi}{2} + \frac{2}{3}k\pi$

3) EQ. LINEARI IN SENO E COSENO:

$a \cdot \sin x + b \cdot \cos x + c = 0, \quad a, b, c \in \mathbb{R}$

- $c = 0 \Rightarrow$ EQ. OMOGENEA

DIVIDO PER $\cos x$: $a \tan x + b = 0$,

$\cos x \neq 0$ CIOÈ $x \neq \frac{\pi}{2} + k\pi$.

- $c \neq 0 \Rightarrow$ EQ. NON OMOGENEA.

→ FORMULE PARAMETRICHE

→ $X := \cos x \qquad Y := \sin x$

$$\begin{cases} a \cdot Y + b X + c = 0 \\ X^2 + Y^2 = 1 \end{cases}$$

Es. (a) $3\sin x + \sqrt{3}\cos x = 0$.

(b) $\sin\left(\frac{5}{6}\pi - x\right) - \cos\left(\frac{4}{3}\pi - x\right) = 2$.

$\sin\left(\frac{5}{6}\pi - x\right) = \sin\left(\frac{5}{6}\pi\right)\cdot\cos(x) - \sin(x)\cdot\cos\left(\frac{5}{6}\pi\right) =$

$= \frac{1}{2}\cdot\cos x + \frac{\sqrt{3}}{2}\cdot\sin x$

(a) $\cos x \neq 0 \iff x \neq \frac{\pi}{2} + k\pi$.

$3\tan x + \sqrt{3} = 0 \iff \tan x = -\frac{\sqrt{3}}{3}$

$\iff x = \frac{5}{6}\pi + k\pi, \quad k \in \mathbb{Z}$.

GUARDO SE $x = \frac{\pi}{2} + k\pi$ È SOL. → NO!

$3\sin x + \sqrt{3}\cos x = 0$

$x = \frac{\pi}{2} \Rightarrow 3\cdot\sin\frac{\pi}{2} + \sqrt{3}\cos\frac{\pi}{2} \overset{?}{=} 0$ NO.

(b) $\sin\left(\frac{5}{6}\pi - x\right) - \cos\left(\frac{4}{3}\pi - x\right) = 2$.

$\cos x + \sqrt{3}\sin x = 2$.

$X := \cos x \qquad Y := \sin x$

$\begin{cases} X + \sqrt{3}Y = 2 \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow \begin{cases} X = 2 - \sqrt{3}Y \\ (2-\sqrt{3}Y)^2 + Y^2 = 1 \end{cases}$

$\Rightarrow \begin{cases} * \\ 4 + 3Y^2 - 4\sqrt{3}Y + Y^2 = 1 \rightarrow 4Y^2 - 4\sqrt{3}Y + 3 = 0 \end{cases}$

$\Delta = 0$

$$\Rightarrow \begin{cases} X = 2 - \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{1}{2} \\ Y = \frac{\sqrt{3}}{2} \end{cases}$$

$$\Rightarrow \begin{cases} \cos x = \frac{1}{2} \\ \sin x = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow x = \frac{\pi}{3} + 2k\pi \quad , k \in \mathbb{Z}$$

4) EQ. DI 2° IN SENO E COSENO:

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x + d = 0$$

$$a, b, c, d \in \mathbb{R}$$

- $d = 0 \Rightarrow$ EQ. OMOGENEA.

$/ \cos^2 x \quad , \quad \cos x \neq 0.$

$$a \tan^2 x + b \tan x + c = 0$$

- $d \neq 0 \Rightarrow$ EQ. NON OMOGENEA.

$$d\left(\cos^2 x + \sin^2 x\right) = d \cdot 1 = d$$

E MI RICONDUCO AL CASO OMOGENEO.

ES. $6 \sin^2 x - \sqrt{3} \sin x \cos x - \cos^2 x = 0$

$/ \cos^2 x$

$$6 \tan^2 x - \sqrt{3} \tan x - 1 = 0$$

$t = \tan x \qquad 6t^2 - \sqrt{3}t - 1 = 0$

$\Delta = 27 \qquad \sqrt{27} = \sqrt{3^3} = 3\sqrt{3}$

$$t_{1,2} = \frac{\sqrt{3} \pm \sqrt{27}}{12} = \begin{cases} \frac{\sqrt{3}+3\sqrt{3}}{12} = \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}-3\sqrt{3}}{12} = \frac{-2\sqrt{3}}{12} = -\frac{\sqrt{3}}{6} \end{cases}$$

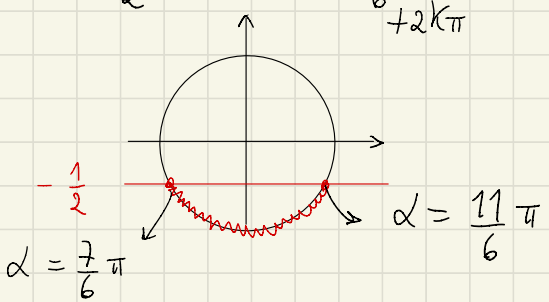
$$\Rightarrow \tan x = \frac{\sqrt{3}}{3} \quad \vee \quad \tan x = -\frac{\sqrt{3}}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + k\pi \quad \vee \quad x = \arctan\left(-\frac{\sqrt{3}}{6}\right) + k\pi, \quad k \in \mathbb{Z}$$

CONTROLLO SE $x = \frac{\pi}{2} + k\pi$ È SOL: NO.

DISEQUAZIONI

1) $\sin x \leq -\frac{1}{2} \iff \frac{7}{6}\pi \leq x \leq \frac{11}{6}\pi$ $+2k\pi$



2) $\sin x \geq -\frac{1}{2}$.

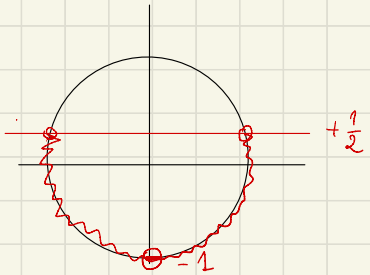
3) $2\sin^2 x + \sin x - 1 < 0$

$t = \sin x$

$2t^2 + t - 1 < 0 \quad \rightarrow$ 

$t_{1,2} = \begin{matrix} -1 \\ 1/2 \end{matrix} \qquad -1 < t < \frac{1}{2}$

$$-1 \leq \sin x < \frac{1}{2}$$



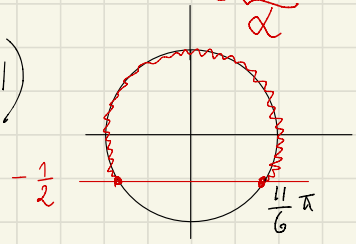
$$\frac{5}{6}\pi < x < \frac{3}{2}\pi$$
$$+2k\pi \qquad +2k\pi$$

$$\rightarrow \frac{3}{2}\pi < x < \frac{\pi}{6} + 2\pi + 2k\pi$$
$$+2k\pi$$

ES. 1) $\sin x \geq -\frac{1}{2}$

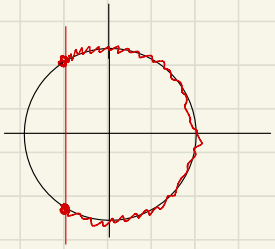
2) $\cos\left(x - \frac{\pi}{6}\right) \geq -\frac{1}{2}$

1)



$$\frac{11}{6}\pi \leq x \leq \frac{7}{6}\pi + 2k\pi \quad , k \in \mathbb{Z}$$
$$-\frac{\pi}{6} + 2k\pi$$

2)



$$\frac{4}{3}\pi \leq \alpha \leq \frac{2}{3}\pi + 2\pi \quad , k \in \mathbb{Z}$$
$$+2k\pi \qquad +2k\pi$$

$$\frac{4}{3}\pi + 2k\pi \leq x - \frac{\pi}{6} \leq \frac{2}{3}\pi + 2\pi + 2k\pi$$

$$\frac{4}{3}\pi + \frac{\pi}{6} + 2k\pi \leq x \leq \frac{2}{3}\pi + 2\pi + 2k\pi + \frac{\pi}{6}$$

$$2k\pi + \frac{9\pi}{6} \leq x \leq \frac{17\pi}{6} + 2k\pi$$