Disequazioni parte 1

DISEQUAZIONI 1° GRADO:

$$x - a > 0 \quad (\geq, <, \leq), \quad a \in \mathbb{R}$$

$$\{x \in \mathbb{R} : x - a > 0\} = \{x \in \mathbb{R} : x > a\} = (a, +\infty)$$

$\geq$ ... $\geq$ ... $[a, +\infty)$

$<$ ... $<$ ... $(-\infty, a)$

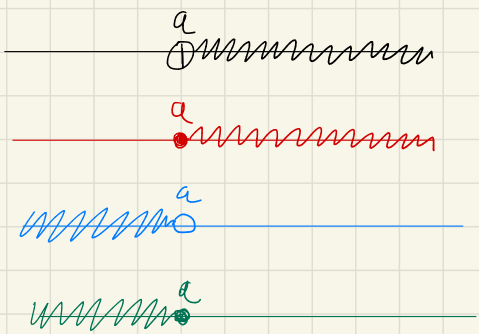
$\leq$ ... $\leq$ ... $(-\infty, a]$

$x > a$: $(a, +\infty)$

$x \geq a$:

$x < a$:

$x \leq a$:



DISEQ. 2° GRADO

$$ax^2 + bx + c > 0 \quad (\geq 0, <, \leq) \quad a, b, c \in \mathbb{R}$$

POSSO SUPPORRE $a > 0$: SE $a < 0$ POTREI MOLTIPLICARE PER (-1) AMBO I MEMBRI.

↘ ATTENZIONE AL VERSO DELLA DISUG.!!

$a = 0 \Rightarrow$ DISEQ. 1° GRADO

$a > 0 \Rightarrow$ $ax^2 + bx + c > 0$ HA LE STESSE SOLUZIONI DI

$$x^2 + \frac{b}{a}x + \frac{c}{a} > 0$$

QUINDI CI SI RICONDUCE AD UNA DISEQ. DEL TIPO:

$$x^2 + bx + c > 0.$$

3 CASI:

1) L'EQ. ASSOCIATA $x^2+bx+c=0$ HA 2 RADICI REALI $x_1 < x_2$.

ALLORA:

$$\{x \in \mathbb{R} : x^2+bx+c>0\} =$$

$$= \{x \in \mathbb{R} : (x-x_1)(x-x_2) > 0\} =$$

$$= \{x \in \mathbb{R} : x-x_1 > 0 \text{ e } x-x_2 > 0\}$$

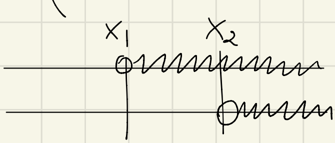
$$\cup \{x \in \mathbb{R} : x-x_1 < 0 \text{ e } x-x_2 < 0\} =$$

$$= \Big(\{x \in \mathbb{R} : x-x_1>0\} \cap \{x : x-x_2>0\}\Big)$$

$$\cup \Big(\{x : x-x_1<0\} \cap \{x : x-x_2<0\}\Big) =$$

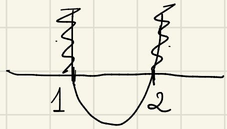
$$= \big((x_1,+\infty) \cap (x_2,+\infty)\big) \cup \big((-\infty,x_1) \cap (-\infty,x_2)\big) =$$

$$x_1<x_2 \Rightarrow = (x_2,+\infty) \cup (-\infty,x_1) = (-\infty,x_1) \cup (x_2,+\infty).$$



ES. $x^2 - 3x + 2 > 0 \rightarrow (x - x_1)(x - x_2) > 0$

$x_1 = 1, \quad x_2 = 2 \qquad (x-1)(x-2) > 0.$



$(-\infty, 1) \cup (2, +\infty).$

ES. $x^2 - 4x - 5 < 0 \qquad (-1, 5)$

ES. $x^2 - 4 > 0 \qquad (-\infty, -2) \cup (2, +\infty).$

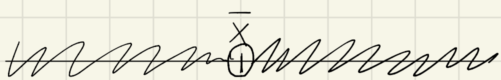
2) L'EQ. ASSOCIATA HA RADICI REALI COINCIDENTI:

$$x_1 = x_2 = \bar{x}.$$

$$x^2 + bx + c > 0 \iff (x - \bar{x})^2 > 0$$

$$\iff x - \bar{x} \neq 0 \iff x \neq \bar{x}.$$

$x \in \mathbb{R} \setminus \{\bar{x}\}.$



ES. $x^2 - 2x + 1 > 0 \iff (x-1)^2 > 0 \iff x \neq 1$

ES. $x^2 - 2x + 1 \geq 0 \iff \forall x \in \mathbb{R}.$

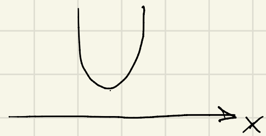
3) L'EQ. ASSOCIATA NON HA RADICI REALI

$$\nexists\, x_1, x_2 \in \mathbb{R} \qquad (\Delta < 0)$$

(L'EQ. ASSOCIATA HA RADICI COMPLESSE CONIUGATE).

$\underline{\text{ES.}}$ $x^2+2x+5>0$

$\Delta<0$



$\forall x\in\mathbb{R}$.

$$x^2+bx+c=\left(x^2+\frac{2bx}{2}\right)+c=$$

$$=\left(x^2+\frac{2bx}{2}+\frac{b^2}{4}\right)+c-\frac{b^2}{4}=$$

$$=\underbrace{\left(x+\frac{b}{2}\right)^2}_{\geq 0}+c-\frac{b^2}{4}\geq 0+c-\frac{b^2}{4}=\frac{4c-b^2}{4}$$

$$x^2+bx+c\geq\frac{4c-b^2}{4}>0.$$

$4c-b^2=-\Delta$

ABBIAMO PROVATO CHE, PER STUDIARE

$(x-p)(x-q)>0$ E' SUFFICIENTE

STUDIARE IL PRODOTTO DEI SEGNI:

		p		q	
$x-p>0$	$-$	0	$+$		$+$
$x-q>0$	$-$		$-$	0	$+$
$(x-p)(x-q)>0$	$+$		$-$		$+$

$p<q$.

$(-\infty,p)\cup(q,+\infty)$.

__Es.__ $(x-1)(x-2)(x-3) \geq 0$

$x-1 \geq 0$
$x-2 \geq 0$
$x-3 \geq 0$

		1		2		3	
$x-1$	$-$		$+$		$+$		$+$
$x-2$	$-$		$-$		$+$		$+$
$x-3$	$-$		$-$		$-$		$+$
	$-$		$+$		$-$		$+$

$x \in [1,2] \cup [3,+\infty)$.

__Es.__ $x^4-16>0$

$$\begin{array}{c} x^4-16 \\ \parallel \\ (x^2-4)(x^2+4) = (x-2)(x+2)(x^2+4) \end{array}$$

$x-2>0 \iff x>2$
$x+2>0 \iff x>-2$
$x^2+4>0 \iff \forall x \in \mathbb{R}$

		-2		2	
$x-2$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$
	$+$		$-$		$+$

$x<-2 \vee x>2$.

$x \in (-\infty,-2) \cup (2,+\infty)$.

Esercizi

1) $x^2 > 8$ 2) $x^2 - 3x < 0$

3) $x^2 + 5x + 1 \geq 0$ 4) $x^2 - 3ax + 2a^2 \leq 0$

5) $(x^2 + x - 2)(x^2 - x - 6) \geq 0$

6) $2x^3 + 3x^2 - 2x - 3 > 0$.

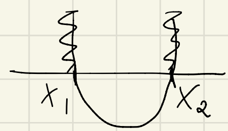
1) $x < -2\sqrt{2} \quad \vee \quad x > 2\sqrt{2}$.

2) $x(x-3) < 0$

	$-$ 0	$+$ 3	$+$
$x > 0$:	$-$	$-$	$+$
$x > 3$:	$+$	$-$	$+$

$$0 < x < 3.$$

3) $\Delta = 21 \qquad x_{1,2} = \dfrac{-5 \pm \sqrt{21}}{2}$



$$x \leq \frac{-5-\sqrt{21}}{2} \quad \vee \quad x \geq \frac{-5+\sqrt{21}}{2}.$$

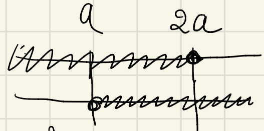
4) $x^2 - 3ax + 2a^2 \leq 0$

$(x-2a)\cdot(x-a) \leq 0$.

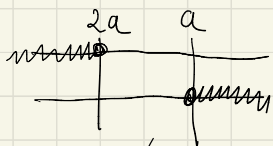
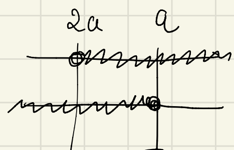
$x = a \Rightarrow a^2 - 3a^2 + 2a^2 = 0$.

$$
\begin{array}{r|l}
x^2-3ax+2a^2 & x-a \\
\hline
\vdots & x-2a \\
\hline
0 &
\end{array}
$$

$\{x : x \le 2a \text{ e } x \ge a\} \cup \{x : x \ge 2a \text{ e } x \le a\}.$

$a > 0$  $[a, 2a] \cup \emptyset$

$a = 0$ $\quad x^2 \le 0 \iff x = 0.$

$a < 0$  $\cup$ 

$\emptyset \quad \cup \quad [2a, a]. \quad = [2a, a].$

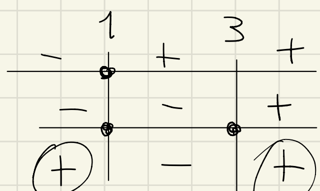
5) $(x^2+x-2)(x^2-x-6) \ge 0$

$\iff (x+2)^2(x-1)(x-3) \ge 0$

$\iff$

$x \ge 1$

$x \ge 3$



$x \in (-\infty, 1] \cup [3, +\infty).$

6) $2x^3 + 3x^2 - 2x - 3 > 0$

$\Leftrightarrow 2x(x^2-1) + 3(x^2-1) > 0$

$\Leftrightarrow (x^2-1)(2x+3) > 0$

$\Leftrightarrow (x+1)(x-1)(2x+3) > 0$

$\Leftrightarrow x \in \left(-\frac{3}{2}, -1\right) \cup (1, +\infty)$.