16/9/22

Logaritmi

$\log_a x \qquad x>0, \quad a>0, \quad a\neq 1.$

SI DIMOSTRA CHE $f(x)=a^x$ PER $a>1$:

- E' STRETT. CRESCENTE SU $\mathbb{R}$
- E' CONTINUA $\forall x\in\mathbb{R}$
- $f(\mathbb{R}) = (0,+\infty)$.

$\Rightarrow \exists$ LA FUNZ. INVERSA $f^{-1}: (0,+\infty)\to\mathbb{R}$

$f(x)=a^x \qquad f^{-1}(x) = \log_a x$

- $f^{-1}(f(x)) = x \quad \forall x\in\mathbb{R}$

  CIOE' $\log_a a^x = x$

- $f(f^{-1}(y)) = y \quad \forall y>0$

  CIOE' $a^{\log_a y} = y$.

<u>PROP.</u> di a^x: 1) $a^x\cdot a^y = a^{x+y}$ 2) $a^1=a$

<u>PROP</u> di $\log_a x$:

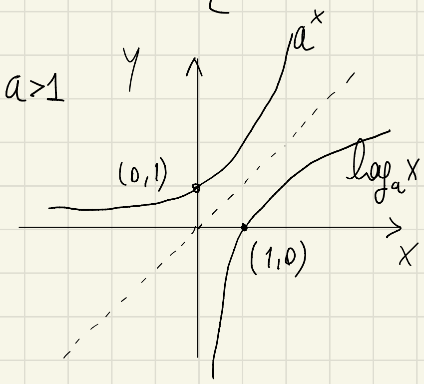
1') $\log_a(x\cdot y) = \log_a x + \log_a y \quad \forall x,y\in(0,+\infty)$.

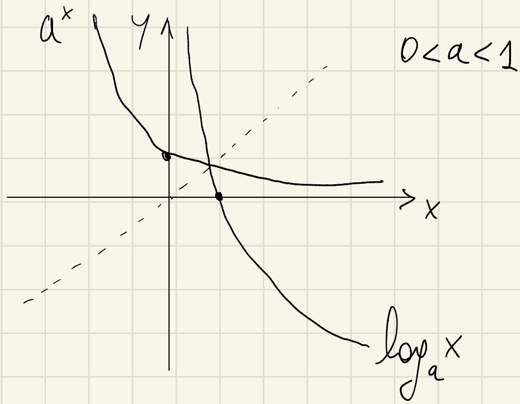
DIM. $f^{-1}(x)+f^{-1}(y) = f^{-1}(x\cdot y) \quad \forall x,y>0$.

$$xy = f(f^{-1}(x))\cdot f(f^{-1}(y)) \overset{1)}{=}$$
$$= f\left(f^{-1}(x)+f^{-1}(y)\right)$$
$$x\cdot y = f\left(f^{-1}(x\cdot y)\right)$$
$$\Rightarrow f\left(f^{-1}(x)+f^{-1}(y)\right) = f\left(f^{-1}(x\cdot y)\right)$$
$$\Rightarrow f^{-1}(x)+f^{-1}(y) = f^{-1}(x\cdot y).$$

2') $\log_a a = 1$.

3') $\log_a x \begin{cases} >0 & \forall x>1 \\ <0 & \forall x\in(0,1) \end{cases}$ SE $a>1$





4') $\log_a(x^y) = y\cdot\log_a x \quad \forall x,y>0$

5') $\log_a x$ STRETT. CRESC. $\forall a>1$

$\log_a x$ STRETT. DECRESCENTE $\forall a,\ 0<a<1$

6') $\log_a 1 = 0 \quad \forall a>0,\ a\neq 1$.

Oss. Le funz. a^x e $\log_a x$ permettono di dare un significato all'espressione

$$x^y, \quad x>0.$$

$$x = a^{\log_a x} \qquad \forall x>0, \ \forall a>0, \ a\neq 1$$

$$x^y = a^{\log_a (x^y)} = a^{y \log_a x} \qquad \forall x>0, \ \forall y \in \mathbb{R}$$

ES.

1) $\log_3 x = 3$

2) $\log_3 x = \log_3 2 - \log_3 (x+1)$.

1) $y = a^x \iff \log_a y = x$.

$x>0$.

$$x = 3^3 = 27.$$

2) $\log_3 x = \log_3 \left(\frac{2}{x+1}\right) \Rightarrow x = \frac{2}{x+1}$

C.E.

$$\begin{cases} \boxed{x>0} \\ x+1>0 \Rightarrow x>-1 \end{cases}$$

$$x(x+1) = 2 \iff x^2+x=2 \iff$$

$$\iff x^2+x-2=0 \iff (x+2)(x-1)=0$$

$$\iff \underbrace{\cancel{x=-2}}_{\text{N.A.}} \ \vee \ x=1.$$

FORMULA CAMBIO BASE.

Siano $a, b > 0$ $\quad a \neq 1,\ b \neq 1,\quad x > 0$:

$$1)\quad \log_a x = \frac{\log_b (x)}{\log_b (a)} \qquad\qquad 2)\quad \log_b x = \frac{\log_a x}{\log_a b}$$

DIM. PER DEF. DI FUNZ. INVERSA:

$$x = a^{\log_a x} = b^{\log_b x} > 0$$

$$\Downarrow$$

$$\log_b x = \log_b \left(a^{\log_a x} \right) \overset{4')}{=} \log_a x \cdot \log_b a$$

$$\Downarrow$$

$$1)$$

ANALOGAMENTE SI OTTIENE 2).

ES. DIM. CHE $\log_a b \cdot \log_b a = 1$. $\quad \forall a, b > 0$, $a, b \neq 1$.

$$\frac{\log_b (b)}{\cancel{\log_b (a)}} \cdot \cancel{\log_b a} = 1 .$$

ES. 1) $\log_2 x + \log_4 x = 3$

2) $\log_2 x + \log_3 x = 1$.

1) $x > 0$

$$\log_2 x + \frac{\log_2 x}{2} = 3 \iff \frac{3}{2}\log_2 x = 3$$

$$\boxed{\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2}}$$

$$\iff \log_2 x = \cancel{3}\cdot\frac{2}{\cancel{3}} \iff x = 2^2 = 4.$$

2) $x > 0.$

$$\log_2 x \cdot \log_3 x = 1$$

$$\log_3 x \cdot \log_2 3 \cdot \log_3 x = 1$$

$$\left(\log_3 x\right)^2 = \frac{1}{\log_2 3} = \log_3 2.$$

$$\log_3 x = \left(\log_3 2\right)^{1/2} \iff x = 3^{\left(\log_3 2\right)^{1/2}}.$$

3) $4\log_4 x - \log_2(1+x) = 0$

4) $\log_x e + \log_e x - 2 = 0$

3) $$\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2}$$

$$\overset{2}{\cancel{4}}\cdot\frac{\log_2 x}{\cancel{2}} - \log_2(1+x) = 0$$

$$\log_2 x^2 - \log_2 (1+x) = 0$$

$$\log_2 \left(\frac{x^2}{1+x}\right) = 0. = \log_2 1$$

$$\frac{x^2}{1+x} = 1. \iff x^2 = 1 + x \iff$$

$$\iff x_{1,2} = \frac{1 \cancel{\pm} \sqrt{5}}{2}.$$ N.A.

4) $\log_x e + \log_e x - 2 = 0$ $\qquad x > 0,\ x \neq 1.$

$$\log_x e = \frac{\log_e e}{\log_e x} = \frac{1}{\log_e x}$$

$$\frac{1}{\log_e x} + \log_e x - 2 = 0$$

$$1 + \ln^2 x - 2 \ln x = 0.$$

$$t := \ln x \qquad t^2 - 2t + 1 = 0 \qquad \log_e x$$

$$(t-1)^2 = 0 \Rightarrow t = 1. \Rightarrow \ln x = 1$$

$$\Rightarrow x = e.$$

DISEQ. LOG.

$a^x > a^y \qquad a > 1 \iff x > y$

$\qquad\qquad\quad 0 < a < 1 \iff x < y.$

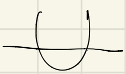
$\log_a x > \log_a y \qquad a > 1 \iff x > y$

$\qquad\qquad\quad 0 < a < 1 \iff x < y.$

ES. $\ln(\underbrace{x^2+5x-6}_{>0}) > \ln(\underbrace{3x^2-3x-6}_{>0})$

C.E. $\begin{cases} x^2+5x-6 > 0 \\ 3x^2-3x-6 > 0 \end{cases}$ $\qquad \Delta = 25+24 = 49$

$x_{1,2} = \dfrac{-5 \pm 7}{2} = \begin{cases} 1 \\ -6 \end{cases}.$



$\leftarrow \begin{cases} (x+6)(x-1) > 0 \\ 3(x^2-x-2) > 0 \end{cases} \qquad \begin{cases} * \\ 3(x-2)(x+1) > 0 \end{cases}$

$\begin{cases} (-\infty,-6) \cup (1,+\infty) \\ (-\infty,-1) \cup (2,+\infty) \end{cases}$

C.E. $(-\infty,-6) \cup (2,+\infty).$

$x^2+5x-6 > 3x^2-3x-6$

$-2x^2+8x > 0 \iff 2x^2-8x < 0$

$\Rightarrow 2x(x-4) < 0 \Rightarrow x \in (0,4)$

$$\left((-\infty,-6)\cup(2,+\infty)\right)\cap(0,4)$$

$$\parallel$$

$$(2,4).$$