Insiemi

DEF. INSIEME: UNA COLLEZIONE DI OGGETTI, I QUALI SONO DETTI ELEMENTI DELL'INSIEME.

RAPPRESENTAZIONE:

1) PER ELENCAZIONE $A=\{2,3,4,5\}$

$B=\{-2,2\}$

2) PER CARATTERISTICA $A=\{x\in\mathbb{N} : 1<x<6\}$

$B=\{x\in\mathbb{Z} : x^2=4\}$

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1) $\in$ (APPARTENENZA) $2\in A$, $1\notin A$

2) $\emptyset$ (INSIEME VUOTO)

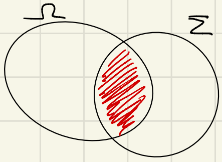
DEF. E' L'INSIEME CHE NON POSSIEDE ELEMENTI, OVVERO $\forall x,\ x\notin\emptyset$.

3) $\subseteq$ (INCLUSIONE)

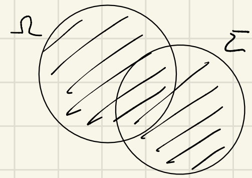
DEF. $\Omega\subseteq\Sigma \iff \forall x\in\Omega,\ x\in\Sigma$

4) $\cap$ (INTERSEZIONE)

DEF. $\Omega\cap\Sigma := \{x : x\in\Omega \text{ e } x\in\Sigma\}$

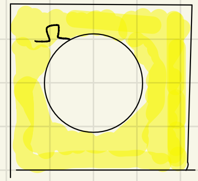


5) $\cup$ (UNIONE)



DEF. $\Omega \cup \Sigma := \{x : \ x \in \Omega \ \text{ o } \ x \in \Sigma\}$

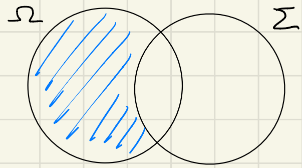
6) Ω^c (COMPLEMENTARE)



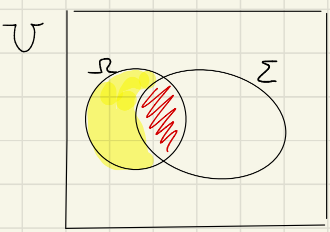
DEF. $\Omega^c \overset{\text{DEF.}}{:=} \{x : \ x \notin \Omega\}$

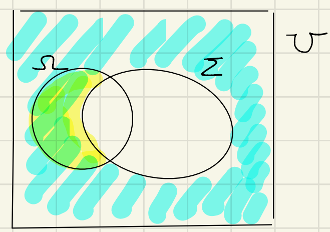
7) $\setminus$ (DIFFERENZA)

DEF. $\Omega \setminus \Sigma := \{x : \ x \in \Omega \ \text{ e } \ x \notin \Sigma\}$.



Oss. • $\boxed{\Omega \setminus \Sigma} = \boxed{\Omega \setminus (\Sigma \cap \Omega)} = \boxed{\Omega \cap \Sigma^c}$





• $(\Omega^c)^c = \Omega$

• $\emptyset^c =: U$ INSIEME AMBIENTE (o UNIVERSO)

Teorema (Leggi di De Morgan)

Dati gli insiemi Ω e Σ

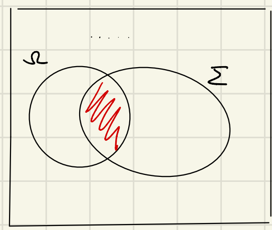
(i) $(\Omega \cap \Sigma)^c = \Omega^c \cup \Sigma^c$

(ii) $(\Omega \cup \Sigma)^c = \Omega^c \cap \Sigma^c$

Dim. (i) $(\Omega \cap \Sigma)^c = \{x : x \in \Omega \text{ e } x \in \Sigma\}^c =$

$= \{x : x \notin \Omega \text{ o } x \notin \Sigma\} = \Omega^c \cup \Sigma^c$

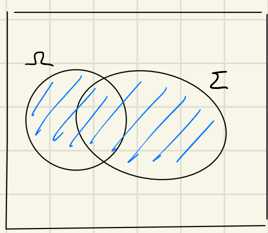
non $(\mathcal{A} \text{ e } \mathcal{B}) =$ (non $\mathcal{A}$) o (non $\mathcal{B}$)



(ii) $(\Omega \cup \Sigma)^c = \{x : x \in \Omega \text{ o } x \in \Sigma\}^c =$

$= \{x : x \notin \Omega \text{ e } x \notin \Sigma\} =$

$= \Omega^c \cap \Sigma^c.$



In generale, per dim. che, dati due insiemi A e B, $A=B$, si può procedere in 2 modi:

1) SFRUTTANDO LE EQUIVALENZE LOGICHE
2) SI PROVA CHE (i) $A \subseteq B$ E (ii) $B \subseteq A$

ESERCIZIO DIMOSTRARE CHE

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

1° MODO: $(\mathcal{A} \text{ e } \mathcal{B}) \text{ o } \mathcal{C} \iff (\mathcal{A} \text{ o } \mathcal{C}) \text{ e } (\mathcal{B} \text{ o } \mathcal{C})$
CHE SI PROVA CON LE TAVOLE DI VERITÀ.

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$	$\mathcal{A}$ e $\mathcal{B}$	($\mathcal{A}$ e $\mathcal{B}$) o $\mathcal{C}$	$\mathcal{A}$ o $\mathcal{C}$	$\mathcal{B}$ o $\mathcal{C}$	($\mathcal{A}$ o $\mathcal{C}$) e ($\mathcal{B}$ o $\mathcal{C}$)
V	V	V	V	V	V	V	V
V	V	F	V	V	V	V	V
V	F	V	F	V	V	V	V
V	F	F	F	F	V	F	F
F	V	V	F	V	V	V	V
F	V	F	F	F	F	V	F
F	F	V	F	V	V	V	V
F	F	F	F	F	F	F	F

$$(A \cap B) \cup C = \{x : x \in C \text{ o } x \in A \cap B\} =$$
$$= \{x : x \in C \text{ o } (x \in A \text{ e } x \in B)\} =$$
$$= \{x : (x \in C \text{ o } x \in A) \text{ e } (x \in C \text{ o } x \in B)\} =$$
$$= (A \cup C) \cap (B \cup C)$$

2° MODO:

(i) $\underbrace{(A \cap B) \cup C}_{=: \Sigma} \subseteq \underbrace{(A \cup C) \cap (B \cup C)}_{=: \Omega}$

Sia $x \in \Sigma$. Devo dim che $x \in \Omega$.

1) $x \in (A \cap B)$ 2) $x \in C$

1) $x \in A \cap B \Rightarrow x \in A$ e $x \in B \Rightarrow$

$\Rightarrow x \in A \cup C$ e $x \in B \cup C \Rightarrow x \in \Omega$.

2) $x \in C \Rightarrow x \in A \cup C$ e $x \in B \cup C \Rightarrow x \in \Omega$.

(ii) Sia $x \in \Omega$. Devo provare che $x \in \Sigma$.

Se $x \in \Omega \Rightarrow x \in A \cup C$ e $x \in B \cup C$

1) $x \in C \Rightarrow x \in C \cup (A \cap B) \Rightarrow x \in \Sigma$.

2) $x \in A,\ x \notin B,\ x \notin C \Rightarrow x \notin \Omega$ quindi non devo procedere

3) $x \in B,\ x \notin A,\ x \notin C$ $\nearrow$

4) $x \in A \cap B,\ x \notin C \Rightarrow x \in (A \cap B) \cup C \Rightarrow x \in \Sigma$.

$\blacksquare$

$\Sigma \subseteq \Omega,\ \Omega \subseteq \Sigma \Rightarrow \Omega = \Sigma$.

DEF. $\mathcal{P}(\Omega)$ (INSIEME DELLE PARTI DI Ω)

È L'INSIEME DEI SOTTOINSIEMI DI Ω,

OVVERO $\mathcal{P}(\Omega) = \{E : E \subseteq \Omega\}$.

Oss. $\emptyset \in \mathcal{P}(\Omega)$, $\Omega \in \mathcal{P}(\Omega)$ $\forall \Omega$.

ESEMPIO: $\Omega = \{a, b, c\} \rightarrow 3$ el.

$$\mathcal{P}(\Omega) = \{\emptyset ; \Omega ; \{a\} ; \{b\} ; \{c\} ; \{a,b\} ; \{a;c\} ; \{b;c\}\}.$$

8 el.

Oss. $|\mathcal{P}(\Omega)| = 2^{|\Omega|}$

$8 = 2^3$ ✓

DEF. PRODOTTO CARTESIANO

$$\Omega \times \Sigma := \{(x,y) : x \in \Omega , y \in \Sigma\}.$$

ESEMPIO

$$\{1, x\} \times \{a, 1, \bullet\} = \{(1,a) ; (1,1) ; (1,\bullet) ; (x,a) ; (x,1) ; (x,\bullet)\}.$$

Esercizi sugli insiemi

Esercizio 1.20 : determinate gli insiemi $\{x \in \mathbb{R} : x^2 \leq 1 \ \mathbf{o} \ x^2 \geq 5\}$ e $\{x \in \mathbb{R} : x^2 < 100 \ \mathbf{e} \ (2x - 1 \leq 0 \ \mathbf{o} \ x > 7)\}$.

Esercizio 1.21 : dite quali fra le seguenti uguaglianze sono vere:

a) $\{x \in \mathbb{R} : (x > 2 \ \mathbf{e} \ x < 6) \ \mathbf{o} \ x < 0\} = \{x \in \mathbb{R} : x > 2 \ \mathbf{e} \ (x < 6 \ \mathbf{o} \ x < 0)\}$

b) $\{x \in \mathbb{R} : (x < 1 \ \mathbf{o} \ x > 3) \ \mathbf{e} \ x \leq 2\} = \{x \in \mathbb{R} : x < 0 \ \mathbf{o} \ (x < 1 \ \mathbf{e} \ x \geq -3)\}$.

Esercizio 1.23 : provate le seguenti formule:

a) $[G \subset E] \iff [\forall F, \ E \cap (F \cup G) = (E \cap F) \cup G]$

b) $E \setminus F = E \cap F^c$

c) $[E \setminus F = \varnothing] \iff [E \subset F]$

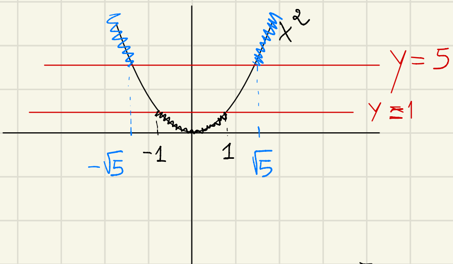
d) $(E \setminus F) \cap (F \setminus E) = \varnothing$.

Esercizio 1.24 : determinate $\mathscr{P}(\{a, 1, \bullet\})$.

✓ **Esercizio 1.25** : determinate tutti gli elementi di $\{1, x\} \times \{a, 1, \bullet\}$.

1) $\{x \in \mathbb{R} : x^2 \le 1 \text{ o } x^2 \ge 5\} = A \cup B$

$A = \{x \in \mathbb{R} : x^2 \le 1\}$ $\quad B = \{x \in \mathbb{R} : x^2 \ge 5\}$.



$A = [-1, 1]$

$B = (-\infty, -\sqrt{5}] \cup [\sqrt{5}, +\infty)$

$A \cup B = (-\infty, -\sqrt{5}] \cup [-1, 1] \cup [\sqrt{5}, +\infty)$.

$\{x \in \mathbb{R} : x^2 < 100 \text{ e } (2x - 1 \le 0 \text{ o } x > 7)\} = A \cap (B \cup C)$

$A = \{x \in \mathbb{R} : x^2 < 100\} = (-10, 10)$

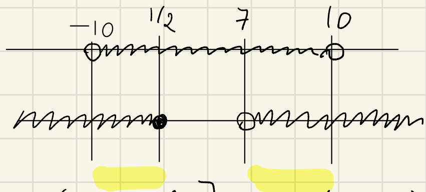
$B = \{x \in \mathbb{R} : 2x - 1 \le 0\} = (-\infty, \frac{1}{2}]$

$C = \{x \in \mathbb{R} : x > 7\} = (7, +\infty)$

$B \cup C = (-\infty, \frac{1}{2}] \cup (7, +\infty)$

A :

$B \cup C$:



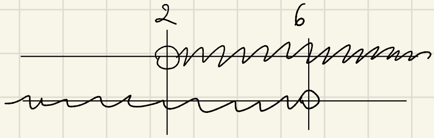
$A \cap (B \cup C) = (-10, \frac{1}{2}] \cup (7, 10)$.

2) (a) $\{x \in \mathbb{R} : (x > 2 \text{ e } x < 6) \text{ o } x < 0\} = \{x \in \mathbb{R} : x > 2 \text{ e } (x < 6 \text{ o } x > 0)\}$

$(A \cap B) \cup C = A \cap (B \cup C)$ $\quad$ FALSA.

$A = (2, +\infty)$ $\quad B = (-\infty, 6)$ $\quad C = (-\infty, 0)$.

$$A \cap (B \cup C) = (2,+\infty) \cap (-\infty, 6) = (2,6)$$



$$\neq$$

$$(A \cap B) \cup C = (2,6) \cup (-\infty, 0)$$

b) VERA

3) a) $E \setminus F = E \cap F^c$

$$E \setminus F = \{x \in \mathbb{B} : x \in E \text{ e } x \notin F\} =$$

$$= \{x : x \in E\} \cap \{x : x \notin F\} =$$

$$= E \cap F^c$$

b) $E \setminus F = \emptyset \iff E \subseteq F$.

$(\Rightarrow)$ $E \setminus F \overset{(a)}{=} E \cap F^c = \emptyset$

$$\Rightarrow \forall x \in E,\ x \notin F^c \Rightarrow \forall x \in E,\ x \in F \Rightarrow E \subseteq F.$$

$(\Leftarrow)$ $E \subseteq F \Rightarrow E \cap F^c = \emptyset \Rightarrow E \setminus F \overset{(a)}{=} E \cap F^c = \emptyset.$